Globalization and Social Segmentation

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Abstract

To analyse the impacts of globalization on social segmentation in advanced economies, we build a model in which (i) households differ in their skill and capital endowments, and (ii) there is a minimal consumption under which they are excluded from the labour market. North-South globalization (NSG) changes income distribution in favour of skilled labour and capital, and North-North globalization (NNG) creates tax competition. The model endogenously generates four types of households: the excluded, the rentiers, the ‘classical’ (whose working time increases with real wages) and the ‘non-classical’ (displaying the opposite relationship). Globalization modifies the size of each group. NNG makes the groups of rentiers and excluded to expand. The impact of NSG is more ambiguous, but it also expands both groups at the outset of globalization. The simulations performed with plausible values of the parameters and factor payments show that Globalization (NSG+NNG) increases the number of excluded and the number of rentiers.

Keywords. Exclusion, Globalization, Rentiers, Social segmentation, Tax competition.

JEL Classification. H2 / J22 / J32 / D31 / D33 / F16

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1. Introduction

We analyse the influence of globalization upon social segmentation by focusing on the changes in income distribution and taxation and on their impacts upon the excluded and the rentiers.

In the mid-eighties, advanced economies had already achieved most of their trade liberalization. Since then, the World has experienced a new and multidimensional globalization process displaying several major characteristics. First, emerging economies (the South) have become key actors of international trade and production. The role of the South has been favoured by trade liberalization in emerging countries and by the strategies of multinational firms that have transferred capital and technologies to less advanced countries. Second, the international mobility of capital has critically grown, and this mobility is now almost perfect across advanced economies (the North). In the North, these two dimensions of globalization have modified income distribution. North-South openness has led to a displacement of income in favour of capital and skill to the detriment of unskilled labour. In addition, capital mobility has supported corporate tax competition between advanced countries, which has raised again the return to capital. By increasing the income share of capital earners and skilled workers at the expense of unskilled workers, these moves in income distribution have typically boosted income inequality. If, until the mid-nineties, the impact of globalization upon income distribution in advanced countries was considered as weak, the subsequent literature has diagnosed a significant impact in most countries, with albeit differences in intensity across them (Chusseau et al., 2008, for a survey).

Along with the concomitant development of globalization and inequality, a number of empirical works have diagnosed an increase in poverty and exclusion in advanced economies (Wolff, 2010, and Antuofermo & Di Meglio, 2012, for Europe; DeNavas-Walt et al., 2013, for the US). In addition, growing inequality and increasing capital share in total income have led to a questioning upon the possible ‘return of the rentiers’. Actually, the XXth century experienced a substantial decline of the rentiers amongst the top incomes (Piketty, 2003; Piketty & Saez, 2003; Piketty et al., 2014). However, the rising income shares of both capital and top incomes coupled with the decrease in corporate taxation, in the top marginal income tax and in inheritance tax render the resurgence of the rentiers rather likely. For Piketty (2014), the increase in the amount of inheritance creates a new group of ‘petit rentiers’.

This article develops a model to analyse the influence of globalization upon social segmentation within a small open advanced economy, by focusing on the impacts on the
excluded and the rentiers. Social segmentation is endogenously generated by the labour supply behaviours of heterogeneous households who differ in skill and capital endowments. By assuming a minimal consumption below which households are excluded from the labour market, we firstly show that the economy is divided between four types of households, namely, the excluded, the rentiers, the ‘classical’ and the ‘non-classical’. Classical households are defined by a labour supply that increases with the real wage whereas the non-classical display the opposite relationship.

To analyse the impact of globalization upon social segmentation, we make a distinction between North-South globalization (NSG) and North-North globalization (NNG). NSG rests upon North-South trade and North-South capital and technology transfers. These transfers make both regions to share the same technology, and North-South trade modifies income distribution in the North in favour of skilled labour and capital at the detriment of unskilled workers. NNG means perfect capital mobility between northern countries, which generates corporate tax competition and thus a downward shift in statutory corporate tax rates.

Globalization modifies the sizes of social groups. The theoretical analysis shows that (i) North-North globalization extends the group of excluded and the group of rentiers, and (ii) North-South globalization has the same impact except when it leads to very large changes in factor payments.

The simulations performed from plausible values of the parameters and of the changes in factor payments confirm that both types of globalization increase the number of excluded and the number of rentiers.

The paper is original in several respects. It firstly endogenously determines social segmentation based on the labour supply behaviour of heterogeneous households. Secondly, it provides theoretical bases for the relation between globalization and changes in the social structure. It finally shows that globalization increases the weights of the groups situated at both extremities of the social spectrum, i.e., the excluded on the one hand and the rentiers on the other hand.

The paper is structured as follows. Section 2 presents a brief review of the literature. Section 3 exposes the bases of the model. Section 4 determines the derived social segmentation and its main characteristics. The effects of globalization on social segmentation is analysed in Section 5. Section 6 provides simulations of these impacts from plausible values of the parameters and of factor payments. The main findings are discussed and we conclude in Section 7.
2. Literature

The model developed in this paper relates to two major strands of literature, namely, the impact of globalization on income distribution and the analysis of social segmentation.

2.1. Globalization and income distribution

There are several channels by which globalization impacts on income distribution. The first and mostly analysed is the impact of North-South openness (trade, offshoring and FDI) upon the skill premium, i.e., inequality between skilled and unskilled workers. Trade openness can also modify income distribution within Melitz-type models and globalization can substantially increase the top incomes. Finally, capital mobility acts through tax competition.

The impact of globalization upon the skill premium in advanced countries has given rise to an abundant theoretical and empirical literature (reviews by Chusseau et al., 2008, and Chusseau & Dumont, 2013). If this impact was considered as weak or negligible until the mid-nineties (Borjas et al, 1992; Katz and Murphy, 1992; Krugman and Lawrence, 1993; Lawrence and Slaughter, 1993), this early diagnosis has subsequently been reconsidered, particularly because of the huge increase in the weight of emerging countries in world trade and production (Krugman, 2008). Empirical works have shown that imports of manufacturing from the South, offshoring to the South and FDI outflows to the South have lessened the demand for unskilled workers and raised the skill premium in the North. In addition, the increase in the share of capital in total income within advanced economies and the decrease in the labour share are now well documented (e.g., Bentolina & Saint-Paul, 2003, CB0, 2011, Karabarbounis & Neiman, 2014).

The literature provides several ways to model the increase in the skill premium and the return to capital that derives from North-South openness. Within a simple neo-classical framework, this can be made from either a one-sector or a multi-sector framework with the North being relatively better endowed with capital and skill and the South with unskilled labour. In these cases, North-South openness leads to an increase in the returns to capital and skill in relation to the payment for unskilled labour in the North. This directly stems from the fact that the passage from North in autarky to North-South openness results in augmenting the unskilled labour supply in relation to both capital and skill. In this vein, a numerous literature has developed Heckscher-Ohlinian frameworks to analyse the impact of North-South trade upon the skill premium and inequality in the North (reviewed in Hellier, 2013).
Another modelling of the relationship between openness and inequality can be found in Melitz-type approaches (Melitz, 2003). By creating export-driven over-profits for the most productive firms, this type of model generates between-firm inequalities and possible changes in income distribution linked to labour market specificities: efficiency wages (Egger & Kreickemeier, 2012; Amiti & Davis, 2011), matching frictions (Helpman et al., 2010), bargaining (Felbermayr et al., 2008) etc. This type of model is however not centred on North-South globalization and it usually does not integrate capital.

Finally, the most recent literature puts forward the huge increase in the share of top incomes in total income.¹ Within an extended HOS model, Haskel et al. (2012) have shown that this can be explained by the impact of globalization when workers differ in talent and when talent and capital are complementary. In Grossman’s model (2004), the interplay between imperfect competition in the labour market and international trade is also beneficial to the most talented.

In the economic literature, the impact of capital mobility upon corporate taxes has been essentially analysed through corporate tax competition (CTC). The basic idea of CTC is that capital mobility incites multinational firms to localise their capital, production and profits in the countries where the corporate tax is low. Consequently, governments are themselves incited to decrease the corporate tax rate so as to attract capital from abroad. This generates a ‘race to the bottom’ between countries in terms of taxation. Following the seminal work of Zodrow & Mieszkowski (1986), the analysis of tax competition has known a large development over the last 25 years, both theoretically and empirically. The major finding of Zodrow & Mieszkowski is that tax competition leads to sub-optimal situations in terms of social welfare characterised by low capital taxation and under-provision of public goods. This result was subsequently extended to different configurations (Wildasin, 1988; Bucovetsky & Wilson, 1991; Kanbur & Keen, 1991; Wilson, 1999 etc.). If the result in terms of optimality is conditioned by the hypothesis of a benevolent public planner, the decrease in the corporate tax rate is a general prediction, except when levies are utilised to improve firms’ profitability (Bénassy-Quéré et al., 2007).

CTC has been tested and estimated in several ways. The results of the empirical literature critically depend on the method and indicators selected to measure corporate taxation. In summary, the CTC hypothesis is confirmed when focusing on strategic interactions (Devereux et al., 2008; Overesch & Rincke, 2011; Zodrow, 2010, for a review), on FDI (De

¹ For the Top 1%, this share grew from 7.7% in 1973 to 18.3% in 2007 in the US (Haskel et al., 2012).
Mooij & Ederveen, 2008, and Devereux & Maffini, 2007, for reviews; recent work by Barrios et al., 2012 and on statutory corporate tax rates (Benassy-Quéré et al., 2007; Cassette & Paty, 2008; Devereux & Fuest, 2012), and it is rejected when accounting for the corporate tax on GDP ratio and for the effective tax rate (Slemrod, 2004; Hines, 2005; Mendoza & Tesar, 2005; Dreher, 2006; Devereux et al., 2008; Devereux & Fuest, 2012). Anyway, the last thirty years have clearly displayed a downward convergence in corporate tax rates across countries.

2.2. Social segmentation

In the recent economic literature on social segmentation, two types of approach can be broadly distinguished. The first starts from an exogenous definition of social stratification and tries to measure the level of stratification and its links with inequality (Yitzhaki & Lerman, 1991; Yitzhaki, 1994; Milanovic & Yitzhaki, 2002; Monti & Santoro, 2011). In the second, social segmentation is endogenously generated. These approaches are centred on social mobility, and on educational and social polarization within intergenerational models of human capital accumulation (Chusseau & Hellier, 2013, for a review). Within a perfectly competitive framework, Becker & Tomes (1979) seminal article predicted that all dynasties converge towards the same human capital and skill in the long term. The same result with albeit a slowdown in the convergence can be shown in the case of imperfections in the credit market (Loury, 1981; Becker & Tomes, 1986). From the nineties, a number of theoretical works have analysed education-based social segmentations. Several factors can generate the emergence of a lasting or permanent group of low-educated persons: credit market imperfections with a fixed cost of education (Galor & Zeira, 1993; Barham et al., 1995); an S-shaped education function (Galor & Tsiddon, 1997); neighbourhood effects (Benabou, 1993, 1996; Durlauf, 1994, 1996); the structure of education systems (Driskill & Horowitz, 2002; Bertocchi & Spagat, 2004; Su, 2004; Chusseau & Hellier, 2011; Brezis & Hellier, 2013).

The impact of globalization upon social segmentation has not been much analysed in the economic literature, except as regards the impact of trade upon the skill level of the population, and thus its division between skilled and unskilled workers.

Since the seminal article of Findlay & Kierzkowski (1983), a number of works have analysed the impact of openness on skill accumulation (Falvey et al., 2008, for a review). Findlay & Kierzkowski endogenised the accumulation of human capital within a HOS model with skilled and unskilled labour. They found that openness boosts education and human capital accumulation in the North, and reduces them in the South. The subsequent extensions of this initial model have led to similar findings (Borsook, 1987; Dinopoulos & Segerstrom,
North-South trade is also skill-enhancing in Grossman & Helpman (1991), Janeba (2003), Falvey et al. (2010). The effect is more ambiguous in Borissov & Hellier (2013) who find that the impact of globalization upon the number of skilled workers in the population is not monotonous. In contrast, several works came to the conclusion that trade can lessen human capital accumulation in the North (Cartiglia, 1997; Eicher, 1999). As the education activity essentially utilises skilled labour, openness reduces human capital accumulation in the North by increasing the skill premium and thereby the cost of education. When assuming credit market imperfections, the negative (positive) effect in the North (South) is magnified (Cartiglia, 1997) and more complex mechanisms were shown by Ranjan (2001, 2003). In summary, the influence of globalization upon the sizes of the skilled and unskilled populations in advanced economies displays several opposite effects, and the total impact depends on their respective weights.

The above literature is typically intergenerational and it focuses thereby on the impact of openness and globalization upon social segmentation in the longer term. In our model, we shall focus on the influence of globalization within a given generation characterised by heterogeneous households.

3. The Model

3.1. General framework

We consider a small open advanced economy. This economy comprises $M$ households.

Each household $i = 1...M$ is endowed with one unit of simple labour, an amount $h_i$ of human capital and an amount $k_i$ of capital. Human capital embodies the different characteristics that determine the individual’s productivity: education, experience, non-cognitive skills, membership of influential networks etc.

Let $w_L$ be the wage per unit of simple labour and $w_H$ the wage per unit of human capital. Then, household $i$’s real wage per unit of time (henceforth household $i$’s unit wage) is $w_i = w_L + w_H h_i$. Her/his wage is $W_i = w_i \times t_i$ with $t_i$ her/his working time. Her/his income from capital is $r_i = r k_i$, with $r$ being the real return to capital.

Both capital and human capital are unevenly distributed across households, and household $i$ is thereby fully identified by the couple of endowments $(h_i, k_i)$. Finally, each household possesses one unit of time s/he can allocate to working and/or leisure.
As the model comprises heterogeneous households, three factors and two types of globalization, we select for the sake of simplicity a one-sector approach. Thus, the world economy produces one good the price of which is 1. Production utilises simple labour $L$, skilled labour $H$ and capital $K$ with the Cobb-Douglas technology $Y = AL^{\alpha_L}H^{\alpha_H}K^{\alpha_K}$, $\alpha_L + \alpha_H + \alpha_K = 1$. With competitive markets, each factor is paid at its marginal productivity and the price of each factor is:

$$w_L = \alpha_L AL^{\alpha_L-1}H^{\alpha_H}K^{\alpha_K}; w_H = \alpha_H AL^{\alpha_L}H^{\alpha_H-1}K^{\alpha_K}; r = \alpha_K AL^{\alpha_L}H^{\alpha_H}K^{\alpha_K-1}$$ (1)

The small open economy hypothesis signifies that the factor quantities that determine the country’s factor prices ($w_L$, $w_H$ and $r$) are those of the World and the country is price-taker.

Let $\bar{c}$ be the minimum consumption level that ensures the minimum health and means from which households have a ‘normal’ social life and can thereby participate in the labour market. The lack of access to certain basic goods and services is a usual definition of exclusion, which thus depends on deprivation (Sen, 2000; Perez-Mayo, 2005; Borooah, 2007; D’ambrosio et al., 2011; Devicienti & Poggy, 2011). This is depicted by the following C.E.S. utility function with deprivation$^2$:

$$u_i = \left(b(c_i - \bar{c})^{\frac{\sigma-1}{\sigma}} + (1-t_i)\frac{\sigma-1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}$$ (2)

with $\sigma > 1$ being the elasticity of substitutions between consumption and leisure, $t_i$ the working time and $(1-t_i)$ the leisure time, $c_i$ the consumption and $\bar{c}$ the consumption under which households are excluded from the labour market.

Household $i$ maximises its utility (2) subject to the usual income constraint and $t_i \leq 1$.

There is a corporate tax on the return to capital the rate of which is $\tau$. This tax is levied directly from the firm in the country of production. The related levies are utilised to provide households with the lump-sum transfer $r_G$. Hence $M \times r_G = \tau \sum_{i=1}^{M} rk_i$, which can be written:

$$r_G = \tau \bar{k}$$

with $\bar{k} = M^{-1}\sum_{i=1}^{M} k_i$ being the average capital per household.

Finally, households $i$’s after-tax total income $I_i$ is:

$$I_i = w_L t_i + (1-\tau) r_i + r_G = (w_L + w_H t_i) t_i + rk_i + \tau r (\bar{k} - k_i)$$

$^2$ The most general form of this type of function was firstly analysed by Pollak (1971) and Wales (1971).
An excluded household is a household that cannot buy the minimal consumption $\tau$ even when working the whole of its disposable time. Hence, since $w_i = w_h$ is household $i$’s highest possible wage, we can establish

**Lemma 1:** The households such that $w_i + (1-\tau)r_i + r_G < \bar{c}$ are excluded from the labour market.

### 3.2. Working time

Consider household $i$ who is not excluded ($w_i + (1-\tau)r_i + r_G \geq \bar{c}$). S/He maximises her/his utility (2) such that $w_i t_i + (1-\tau)r_i + r_G \geq c$ and $t_i \geq 0$. This provides the following supply of working time (see Appendix 1):

$$t_i = \max \left\{ \frac{\left(bw_i\right)^{\tau} - (1-\tau)r_i - r_G + \bar{c}}{w_i + \left(bw_i\right)^{\tau}}, 0 \right\} \quad (3)$$

**Lemma 2.** Consider working household $i$. Her/his working time $t_i$:

1) decreases with the return to capital $r$, with the household’s capital endowment $k_i$ and with the average capital endowment $\bar{k}$;

2) decreases with the corporate tax rate $\tau$ if $k_i < \bar{k}$ and increases with $\tau$ if $k_i > \bar{k}$;

3) decreases with the unit wage $w_i$ if $w_i < \hat{w}$ and increases with $w_i$ if $w_i > \hat{w}$, $\hat{w} = \hat{w}(r,k_i,\tau)$ being a function such that $\partial \hat{w} / \partial r < 0$, $\partial \hat{w} / \partial k_i < 0$ and $\partial \hat{w} / \partial \tau > 0$ if $k_i \geq \bar{k}$.

**Proof.** Appendix 2.

An increase in non-labour incomes reduces labour supply because it lessens the incentive to work. As a consequence, an increase in the return to capital $r$ reduces labour supply because it raises both the after-tax private rents $(1-\tau)rk_i$ and the social transfers to the household $r_G = \tau rk$. A rise in the corporate tax $\tau$ lowers the labour supply of households who are poorly endowed with capital ($k_i < \bar{k}$) because this raises their total rents through the public transfers. In contrast, those who possess a rather large amount of capital ($k_i > \bar{k}$) suffer a decrease in their total rents, which incites them to work more.
Finally, there is a wage threshold \( \hat{w} = \hat{w}(r, k_i, \tau) \) below which the working time \( t_i \) is a decreasing function of wage \( w_i \) and above which \( w_i \) increases \( t_i \). In other words, the income effect dominates the substitution effect when \( w_i < \hat{w} \) and the substitution effect dominates the income effect when \( w_i > \hat{w} \). This result directly stems from the hypothesis of a minimum consumption necessary to participate in the labour market. When \( w_i < \hat{w}_i \), the income is low and the household must allow a large part of her/his available time to working so as to go beyond the minimum consumption \( \bar{c} \). Then, a decrease in the wage per unit of time \( w_i \) incites the household to work more so as to maintain her/his income above \( \bar{c} \). In contrast, \( w_i < \hat{w}_i \) corresponds to a situation in which the household's income is comfortably above the minimum consumption \( \bar{c} \). Then, an increase in the unit wage \( w_i \) is necessary to incite the household to work more.

### 4. Social Segmentation

#### 4.1 Types of households

**Definition 1.** We call:

1) **Excluded** the households who cannot attain the minimum consumption \( \bar{c} \) even when working during the whole of their disposable time;

2) **Rentiers** the households who are not excluded and choose not to work;

3) **Classical** the working households whose labour supply increases with their unit wage;

4) **Non-classical** the working households whose labour supply decreases with their unit wage.

It can be noted that the rentiers are not limited to very rich households whose capital income is so high that they prefer not to work. They gather all the households who can live without working and whose potential wage is not high enough to incite them to go to work. In particular, a number of valid retired workers belong to this category: their efficiency has decreased because of skill obsolescence (and presumably loss of dynamism) and their rents are high enough to convince them to move out of work.
**Proposition 1**: Households are distributed between four groups:

1) the excluded are such that \( k_i < E(h_i) \),
2) the non-classical are such that \( E(h_i) \leq k_i < C(h_i) \),
3) the classical are such that \( C(h_i) < k_i < R(h_i) \),
4) the rentiers are such that \( k_i \geq R(h_i) \),

with:

\[
E(h_i) = \frac{c - r_G - w_i}{(1 - \tau)r}; \quad C(h_i) = \frac{c - r_G}{(1 - \tau)r} - \frac{(\sigma - 1)b^\sigma w_i^\sigma}{\left(\sigma b^\sigma w_i^\sigma + 1\right)(1 - \tau)r}; \quad R(h_i) = \frac{c - r_G + b^\sigma w_i^\sigma}{(1 - \tau)r}, \quad w_i = w_L + w_H h_i
\]

Proof. Appendix 3.

We can thus determine the space corresponding to each social group within the quadrant \((h_i, k_i)\). Actually, Proposition 1 shows that the line \( k_i = E(h_i) \) separates the set of excluded from the set of non-classical, the curve \( k_i = C(h_i) \) the set of non-classical from the set of classical households, and the curve \( k_i = R(h_i) \) the set of classical from the set of rentiers.

### 4.2 Social spaces

We assume that individuals are distributed in the interval \([0, h_{\text{max}}]\) in terms of human capital and \([0, k_{\text{max}}]\) in terms of capital. The space \([0, h_{\text{max}}] \times [0, k_{\text{max}}]\) is called ‘Space of households’.

Figure 1 depicts each social space within the space of households. The values \((k_E, k_C, k_R, h_E, h_C, h_R)\) are described in Appendix 4.

*Figure 1. The four social spaces in the Space of households.*
The dimension of each social space, defined as the surfaces of each space in the plan \((h,k)\), is depicted in Table 1 (calculations in Appendix 4).

**Table 1. Social Spaces Dimensions**

<table>
<thead>
<tr>
<th>Spaces</th>
<th>Dimension in the plan ((h_i,k_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space of exclusion</td>
<td>(S_E = \frac{(\bar{c} - \tau r \bar{k} - w_L)^2}{2(1-\tau)rw_H})</td>
</tr>
<tr>
<td>Space of rentiers</td>
<td>(S_R = k_{max} h_R - \frac{b^\sigma \left( (w_{L} + w_{H} h_R)^{\sigma+1} - w_L^{\sigma+1} \right)}{(1-\tau)r(\sigma+1)w_H} - \bar{c} - \tau r \bar{k} - h_R)</td>
</tr>
<tr>
<td>Non-classical households</td>
<td>(S_{NC} = \frac{1}{(1-\tau)r} \int_{0}^{h_c} \left( \bar{c} - r_G - \frac{(\sigma - 1)b^\sigma (w_{L} + w_{H} h)^{\sigma}}{\sigma b^\sigma (w_{L} + w_{H} h)^{\sigma+1} + 1} \right) dh - S_E)</td>
</tr>
<tr>
<td>Classical households</td>
<td>(S_C = k_{max} h_{max} - (S_R + S_{NC} + S_E))</td>
</tr>
</tbody>
</table>

It must be noted that the social spaces dimensions give no information about the proportion of households inside each space, which depends on the distribution of human and physical capital between households. Finally, even if we focus on the excluded and the rentiers, the distinction between classical and non-classical households is crucial because (i) more excluded comes from non-classical households falling into exclusion, and (ii) more rentiers derives from the decision of certain classical to stop working.

4.3. Incomes, corporate tax and social segmentation

We shall henceforth introduce the following five realistic assumptions:

A1. The space of exclusion does exist, i.e., \(w_L + \tau r \bar{k} < \bar{c}\).

A2. All excluded households have a capital endowment lower than the average capital endowment: \(k_i < \bar{k}\). The social transfer they receive is thus higher than the levies they pay, i.e., their rents \((1-\tau)r k_i + \tau r \bar{k} = r k_i + \tau r(\bar{k} - k_i)\) increase with the tax rate \(\tau\).

A3. All the rentiers have a capital endowment higher than the average \(k_i > \bar{k}\), which signifies that \(r \bar{k} < (bw_i)^\sigma + \bar{c}, \ \forall i \in S_R\).

A4. The average income is higher than the minimal consumption \(\bar{c}\).

A5. \(\bar{c} < w_L + (2-\tau)r \bar{k}\). The justification of this assumption is given in Appendix 5.

We now analyse the impact on each social space of the four determinants of the after-tax income, i.e., the return to capital \(r\), the corporate tax rate \(\tau\) and the wages \(w_H\) and \(w_L\).

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3 It is only if households are uniformly distributed in the space of households that dividing each dimension by \(k_{max} \times h_{max}\) provides the exact proportion of households inside the corresponding space.
**Lemma 3.** An increase (decrease) in the return to capital $r$:

1) expands (reduces) the space of the rentiers, and
2) reduces (expands) the space of exclusion.

*Proof:* Appendix 5.

The increase in capital income expands the space of rentiers because it reduces the capital owners’ incentive to work. In addition, the increase in $r$ augments the redistribution to the excluded, which makes some of them escape from exclusion.

**Lemma 4.** An increase (decrease) in the corporate tax rate $\tau$ reduces (expands) the space of rentiers and the space of exclusion.

*Proof:* Appendix 5.

Let us focus on the case in which $\tau$ decreases, which characterises North-North globalization (see below). The decrease in $\tau$ expands the space of rentiers because it rises the rents. The decrease in $\tau$ also increases the space of excluded because it cuts redistribution.

**Lemma 5.** An increase (decrease) in the unit wages $w_L$ and $w_H$ reduces (enlarges) the space of rentiers and the space of exclusion.

*Proof:* Appendix 6.

A rise in wages reduces the space of rentiers because it raises the incentive to work, and it reduces the number of excluded by increasing earnings.

As regards the spaces of classical and non-classical households, the effects of changes in $w_H$, $w_L$, $r$ and $\tau$ depend on the initial factor payments and tax $(\bar{w}_H, \bar{w}_L, \bar{r}, \bar{\tau})$ and on the model parameters $(b, \sigma, \alpha)$. These impacts are simulated in Section 6.

5. Globalization and Social Segmentation

5.1. Globalization

We make a distinction between North-South globalization and North-North globalization.

5.1.1. North-South Globalization (NSG)

North-South globalization is characterised by three features:

1. Free trade between the two areas, with the size of the South increasing throughout the globalization process (Hellier & Chusseau, 2010, and Borissov & Hellier, 2013, for models
with this assumption). The growing size of the South depicts the well-documented increase in the number of emerging countries and regions participating in the globalized economy.

2. Capital and technological transfers from the North to the South, which is a clear result of multinational firms’ FDI.

3. Compared to the North, the South is assumed to display a high relative endowment of simple labour in relation to both skill and capital.

North-South openness thus results in:

1. The adoption by the South of the northern technology\(^4\), and
2. An increase in the world endowment of \(L\) in relation to both \(H\) and \(K\) and thus by a change in the factor payments \(w_H, w_L\) and \(r\).

We assume to simplify that this causes both factor endowments ratios \(L/H\) and \(L/K\) to be multiplied by the same growing coefficient \(\lambda > 1\) at the world level. Because of the Cobb-Douglas technology, the wage per unit of simple labour \(w_L\) is then multiplied by \(\lambda^{\alpha_L - 1}\), the return to skill \(w_H\) and the return to capital \(r\) by \(\lambda^{\alpha_L}\), and the price of the good remains equal to 1 (equations (1)\(^5\)).

We can thus model the increase in the size of the South which defines NSG by an increase in parameter \(\lambda\) from an initial value \(\lambda = 1\). The real wage per unit of simple labour \(\times\) time is \(w_L = \lambda^{\alpha_L - 1} \bar{w}_L\), the real wage per unit of skill \(\times\) time \(w_H = \lambda^{\alpha_L} \bar{w}_H\) and the real unit return to capital \(r = \lambda^{\alpha_L} \bar{r}\), with \(\bar{w}_L, \bar{w}_H\) and \(\bar{r}\) being these values at the outset of globalization. The real lump sum redistribution benefit with NSG is \(r_G = \lambda^{\alpha_L} \bar{r} \bar{k}\).

We determine the minimum skill from which NSG increases the unit wage in the North:

**Lemma 6.** North-South globalization increases (lowers) the unit wage \(w_i\) of the households with a human capital \(h_i\) higher (lower) than \(\bar{h}(\lambda)\), with:

\[
\bar{h}(\lambda) = \frac{1 - \alpha_L}{\alpha_L} \frac{\bar{w}_L}{\bar{w}_H} \lambda^{-1}
\]

*Proof.* \(w_i = \bar{w}_L \lambda^{\alpha_L - 1} + \bar{w}_H \lambda^{\alpha_L} \Rightarrow \partial w_i / \partial \lambda = (\alpha_L - 1)(\bar{w}_L \lambda^{\alpha_L - 2}) + \alpha_L \bar{w}_H \lambda^{\alpha_L - 1}\). Hence:

\[
\partial w_i / \partial \lambda > (\lambda < 0) \Leftrightarrow h_i > (\lambda <) \bar{h}(\lambda) = (1 - \alpha_L) \bar{w}_L / \lambda \alpha_L \bar{w}_H.
\]

\(^4\) The TFP can however remain lower in the South without changing the model’s outcomes.

\(^5\)\(w_i = \alpha_L \lambda^{\alpha_L - 1} H^{\alpha_I} K^{\alpha_K} = \alpha_L \lambda^{\alpha_L - 1} (L/H)^{\alpha_L} (L/K)^{\alpha_K} \Rightarrow w_i(\lambda) = \alpha_L \lambda^{\alpha_L - 1} (L/H)^{\alpha_L} (L/K)^{\alpha_K} = \lambda^{\alpha_L - 1} w_i\)

\(w_H = \alpha_H \lambda^{\alpha_L} H^{\alpha_I} K^{\alpha_K} = \alpha_H \lambda^{\alpha_L} (L/H)^{\alpha_L} (L/K)^{\alpha_K} \Rightarrow w_H(\lambda) = \lambda^{\alpha_L} w_H\). Same demonstration for \(r\).
Lemma 6 shows that NSG divides households into two sets depending on their human capital endowment. Households with a human capital higher than \( h(\lambda) \) benefit from an increase in earnings whereas those with a human capital below \( h(\lambda) \) suffer a decrease.

In addition, NSG (rise in \( \lambda \)) lowers the threshold \( h(\lambda) \) under which globalization reduces earnings \( (\partial h / \partial \lambda < 0) \). As a consequence, a household located under threshold \( h(\lambda) \) at the outset of globalization can display a U-shaped variation of its earnings as \( \lambda \) rises.

5.1.2. North-North globalization (NNG)

NNG is characterised by perfect capital mobility between northern countries resulting in tax competition and thereby in a reduction in the corporate tax rate. As underlined in introduction, the decrease in the statutory corporate tax due to capital mobility is a general result of both the theoretical and empirical literature on corporate tax competition.

The reduction in the corporate tax rate that defines the NNG dynamics will be modelled by an increase in parameter \( \eta \) from the initial value 1, with \( \tau = \tau / \eta \) being the corporate tax rate and \( \tau \) this rate at the outset of globalization. Consequently, \( r_G = \lambda^\alpha \tau k / \eta \) when NNG acts alone and \( r_G = \lambda^\alpha \tau k / \eta \) when NSG and NNG act together.

5.2. Social segmentation

We study the impact of globalization upon social segmentation. We successively analyse the impact of NSG, of NNG and of the combination of both types of globalization.

5.2.1. North-South Globalization

Lemma 7. The space of exclusion:

1) increases with the size of the South \( \lambda \) when \( \lambda < (\bar{W}_L / \alpha c)^{(1 - \alpha L)} \), and

2) decreases with the size of the South when \( \lambda > (\bar{W}_L / \alpha c)^{(1 - \alpha L)} \).

Proof: Appendix 7.

From Lemma 7, we can state the following:

Proposition 2. The relation between the dimension of the space of exclusion \( S_E \) and the size of the South \( \lambda \) has an inverted-U shape.

Proof: Appendix 7.
The inverted-U shape of the relationship that binds the number of excluded to North-South globalization is both logical and mechanical. NSG causes an increase in $r$ and $w_H$, and a decrease in $w_L$. As the incomes of the excluded as well as those of the poorest non-classical households essentially come from $w_L$, NSG firstly lessens these incomes and make the poorest non-classical households fall into exclusion. However, with the simultaneous rise in $w_H$ and $r$ and reduction in $w_L$, a moment comes when these moves make the income of the most skilled (and capital owning) excluded to increase. Then, the rise in $\lambda$ allows a growing number of excluded to attain the minimum consumption $\bar{c}$, which makes them escape from exclusion.

**Proposition 3. North-South globalization:**

1) increases the dimension of the space of rentiers and the number of rentiers at the outset of the NSG process;
2) has subsequently an ambiguous impact upon the dimension of the space of rentiers and the number of rentiers.

North-South globalization has several opposite impacts upon the space of rentiers. A rise in the wage $w_i = w_L + w_H h_i$ lessens the number of rentiers whereas increases in private rents $rk_i$ and in net public rents $r(\bar{k} - k_i)$ augment it. Consequently, the decrease in $w_L$ and the increase in $r$ enlarge the number of rentiers, whereas the increase in $w_H$ shrinks it.

**5.2.2. North-North Globalization (NNG)**

NNG is modelled as a decrease in the country’s corporate tax $\tau$. From Lemma 4, we infer

**Proposition 4. North-North globalization expands the space of exclusion and the space of rentiers.**

Finally, the impacts of NSG (increase in $\lambda$) and NNG (decrease in $\tau$) upon the dimensions of the spaces of classical and non-classical households cannot be analysed in a simple way. They depend on the set of initial values $(\bar{w}_H, \bar{w}_L, \bar{r}, \bar{\tau})$, on the model parameters $(b, \sigma, \alpha_L)$, and on the intensity of the shifts in $\lambda$ and $\eta$. These impacts will be simulated in Section 6 from plausible values of the parameters and of factor payments.
5.2.3. Total impact of Globalization

It is not possible to provide a simple analytical analysis of the impact of the combination of NSG and NNG upon each social space. This is due to the multiple dimensions of globalization and the complexity of their combined effect upon the households according to the share in their total gain of each type of income (wages for simple labour and human capital, capital income, social benefit), and thus according to their social group. The analysis will thereby be implemented in Section 6 by simulating different dynamics corresponding to plausible values of the income shares and the model parameters. From the above results of NSG and NNG, it is however possible to analyse the effects of globalization upon the space of exclusion and the space of rentiers.

**Proposition 5.** Globalization (NSG+NNG) increases the space of exclusion as long as 
\[
\lambda \leq \left( \frac{\bar{w}_L}{\alpha_t \bar{c}} \right)^{1/(1-\alpha)}
\] 
and it has an ambiguous impact on this space when 
\[
\lambda > \left( \frac{\bar{w}_L}{\alpha_t \bar{c}} \right)^{1/(1-\alpha)}.
\]

**Proof.** From Lemma 7 and Proposition 4.

When \( \lambda < \left( \frac{\bar{w}_L}{\alpha_t \bar{c}} \right)^{1/(1-\alpha)} \), both NSG and NNG increase the space of exclusion. When \( \lambda \) becomes higher than \( \left( \frac{\bar{w}_L}{\alpha_t \bar{c}} \right)^{1/(1-\alpha)} \), NSG and NNG have opposite impacts on the space of exclusion (Lemma 7 and Proposition 4). From then, it can be shown (available from the authors upon request) that for each couple of values \((\lambda, \eta)\) there is a minimum rate of increase in \( \eta \), depending on the rate of increase in \( \lambda \), from which the space of exclusion expands. In other words, for the space of exclusion to continue expanding as \( \lambda > \left( \frac{\bar{w}_L}{\alpha_t \bar{c}} \right)^{1/(1-\alpha)} \), the exclusion-enhancing decrease in the corporate tax must be sufficiently large to offset the decrease in the number of excluded due to NSG.

**Proposition 6.** Globalization (NSG+NNG) increases the dimension of the space of rentiers and the number of rentiers at the outset of the globalization process and has subsequently an ambiguous impact upon both.

**Proof.** From propositions 3 and 4.
6. Simulations

Two series of simulations are implemented. Both utilise the same values of the parameters, of the limit values $h_{\max}$ and $k_{\max}$, and of the initial factor payments.

The first set of simulations aims at illustrating the main findings of the theoretical approach. In this purpose, we (i) draw the four social spaces, (ii) calculate the dimension of each space before globalization ($\lambda = \eta = 1$), and (iii) analyse the impacts of NSG and NNG upon these dimensions by making $\lambda$ and $\eta$ vary. As already noted, these calculations cannot portray the globalization-driven changes in the weights of each social group because these weights depend on the distribution of individuals inside the space of households, and this distribution is typically not uniform.

The second series of simulations analyse the impacts of globalization upon the social groups from a distribution of households in the space $(h, k)$ that broadly corresponds to what was observed in the US, which is the only country for which we have indications on the crossed distribution of earnings and capital incomes for households.

6.1. Parameters, initial values and globalization indicators

Table 2 depicts the values of the parameters, the upper limits $h_{\max}$ and $k_{\max}$, the pre-globalization factor payments ($\bar{w}_L$, $\bar{w}_H$, $\bar{\tau}$) and corporate tax ($\tau$). These values are utilised for both series of simulations.

<table>
<thead>
<tr>
<th>$\bar{w}_H$</th>
<th>$\bar{w}_L$</th>
<th>$\bar{\tau}$</th>
<th>$\tau$</th>
<th>$h_{\max}$</th>
<th>$k_{\max}$</th>
<th>$\bar{k}$</th>
<th>$\bar{c}$</th>
<th>$b$</th>
<th>$\sigma$</th>
<th>$\alpha_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>0.03</td>
<td>0.3</td>
<td>10</td>
<td>10000</td>
<td>146.7</td>
<td>4</td>
<td>0.6</td>
<td>2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The values selected for $\bar{w}_L$, $\bar{w}_H$ and $h_{\max}$ make the earnings multiplier between the least skilled ($h = 0$) and the most skilled ($h_{\max} = 10$) household to be 11. The value $\alpha_L = 0.2$ signifies that simple labour accounts for 20% of total income. The initial corporate tax rate $\tau = 0.3$ is between the present rates (which are of about 20-25%) and the rates of the early eighties (about 40-50%). Coefficient $b$ is selected to have a little more than 90% of the disposable time (equal to 1) to be allocated for working in the case of a household with the highest skill ($h = 10$) and no capital ($k = 0$). The minimal consumption $\bar{c}$ is such that the
space of exclusion does exist ($\bar{w}_L + \bar{r}_G < \bar{c}$) and the average capital ($\bar{k} = 146.7$) such that redistribution ($\tau \bar{k} = 1.32$) is lower than the unit wage $\bar{w}_L$ of a household without any skill ($h = 0$). Finally, the same simulations were carried out with different values of the parameters ($\sigma$ varying from 1 to 3, $\tau$ from 0.1 to 0.5, $\varpi$ from 0.01 to 0.05, $\alpha_L$ from 0.15 to 0.35, different values of $b$). All these simulations provide similar outcomes in terms of variation, with however differences in intensity.

We introduce globalization by making $\lambda$ vary from 1 to 1.2, and $\eta$ from 1 to 1.5. The variation in $\lambda$ corresponds to increases in $w_H$ and $r$ by 5.6%, a decrease in $w_L$ by 12%, and an increase in $w_H / w_L$ by 20%. These amounts are in line with the empirical literature on the subject, in which NSG increases the return to skill and diminishes the wage of simple (unskilled) labour. The change in $\eta$ from 1 to 1.5 corresponds to a shift in the redistributive component of the corporate tax rate $\tau$ from 30% down to 20%, which is again a rather limited change. These values have been selected to analyse the impact of globalization on the social structure even when its distributional effects remain limited.

6.2. Social spaces dimensions

6.2.1. Overview

<table>
<thead>
<tr>
<th></th>
<th>$S_E$</th>
<th>$S_R$</th>
<th>$S_{NC}$</th>
<th>$S_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Globalization</td>
<td>0.0055</td>
<td>69.12</td>
<td>0.11</td>
<td>30.76</td>
</tr>
<tr>
<td>NSG</td>
<td>0.009</td>
<td>69.41</td>
<td>0.105</td>
<td>30.48</td>
</tr>
<tr>
<td></td>
<td>(+63.6)</td>
<td>(+41.0)</td>
<td>(-3.5)</td>
<td>(-0.92)</td>
</tr>
<tr>
<td>NNG</td>
<td>0.013</td>
<td>74.54</td>
<td>0.13</td>
<td>25.31</td>
</tr>
<tr>
<td></td>
<td>(+228.4)</td>
<td>(+7.8)</td>
<td>(+21.6)</td>
<td>(-91.8)</td>
</tr>
<tr>
<td>NSG + NNG</td>
<td>0.018</td>
<td>74.73</td>
<td>0.127</td>
<td>25.13</td>
</tr>
<tr>
<td></td>
<td>(+224.9)</td>
<td>(+8.1)</td>
<td>(+17)</td>
<td>(-18.3)</td>
</tr>
</tbody>
</table>

*Between brackets: change in % in relation to the pre-globalisation situation.*

Table 3 provides (i) the dimensions of each space ($S_E, S_R, S_{NC}, S_C$) in percent of the dimension of the space of household (100,000) in the following four cases: 1) before globalization ($\lambda = \mu = 1$); 2) at the end of North-South globalization only ($\lambda = 1.2 ; \eta = 1$); 3) at the end of North-North globalization only ($\lambda = 1 ; \eta = 1.5$); 4) at the end of combined NSG and NNG ($\lambda = 1.2 ; \eta = 1.5$).

As expected, both NSG and NNG enlarge the space of rentiers and the space of exclusion. In addition, both shrink the space of classical households.
It must be noted that, if the spaces of rentiers and of classical households are apparently much bigger than the other spaces, this does not depict the weight of each type of households. Actually, in the real economy, a large majority of households are concentrated in the South-West part of the space \{(h,k)\} and the percentage of households in the space of rentiers is very small whereas the space of non-classical is rather large (see section 6.2).

6.2.2. North-South globalization

Figure 2 depicts the NSG-driven changes in the social spaces dimensions when \(\lambda\) moves from 1 to 1.2.

As expected, NSG increases both the space of excluded and the space of rentiers.

Note that, if we make \(\lambda\) increase beyond 1.2, the curves display an inverted-U shape (as expected for the excluded) with the turning point occurring for \(\lambda = 2.8\) in the case of the space of exclusion, and 1.3 for the space of rentiers (see Appendix 8).

Both spaces of non-classical and classical households shrink. These results are verified for a large range of simulations implemented by making the parameters, factor payments and limit values to vary within plausible intervals.

![Figure 2. NSG and the Social spaces dimensions](image)

In terms of rate of variation (Table 3), with \(\lambda\) moving from 1 up to 1.2, the increase in \(S_E\) is the highest (+63.6%) and the rate of decrease in \(S_C\) remains rather modest (-0.92%).
6.2.3. North-North globalization

Figure 3. NNG and the Social spaces dimensions

Figure 3 draws the variations in the dimension of each space that derive from NNG. The dimensions and limits of each space at the end of NNG ($\eta = 1.5$) are in Table 3.

As expected, the space of exclusion and the space of rentiers expand. In addition the space of non-classical expands as well, which reveals the negative impact of the decrease in redistribution upon the poorest classical who now increase their working time to maintain their post-tax and redistribution income.

6.2.4. Combined NSG and NNG

We now make the couple $(\lambda, \eta)$ to vary from (1,1) to (1.2, 1.5) so as to combine North-South and North-North globalization. Figure 4 depicts the related changes in the social spaces dimensions.

Logically, both the space of exclusion and the space of rentiers expand. In addition, the space of non-classical increases too, showing that the positive effect of NNG dominates the negative effect of NSG. The extension of the space of rentiers combined with the increase in the space of non-classical shows that globalization makes certain classical households to become non-classical. In other words: the former poorest classical have become non-classical; they now increase (decrease) their working time when their unit wage lessens (augments).
6.3. Changes in the households’ positions

We assume 1000 households distributed in the space \([0, h_{\text{max}}] \times [0, k_{\text{max}}]\) with \(h_{\text{max}} = 10\) and \(k_{\text{max}} = 10000\), and we make \(\lambda\) vary from 1 to 1.2 and \(\eta\) from 1 to 1.5. The model parameters and the initial values are the same as in the preceding simulations.

If the distribution by percentile of both labour incomes and financial wealth taken separately can be found for a large range of countries, the crossed distribution is typically not available. For the US, we however have the distribution of wealth per earnings level (intervals) with the weight of each earning interval in total earnings (Wolff, 2012, p.80). We thus build a crossed distribution earnings \(\times\) financial wealth based upon the distributions in the US as revealed by the OECD (for earnings) and Wolff (2012) for financial wealth. This distribution corresponds to an inequality-oriented country.

6.3.1. Changes in the weight of each social group

Table 4 depicts the share of each social group in the population at the initial time \((\lambda = \eta = 1)\), at the end of NSG acting alone \((\lambda = 1.2; \eta = 1)\), at the end of NNG acting alone \((\lambda = 1; \eta = 1.5)\) and when NSG and NNG are combined \((\lambda = 1.2; \eta = 1.5)\).
Table 4. Share of each social group

<table>
<thead>
<tr>
<th></th>
<th>Pre-globalization</th>
<th>NSG alone</th>
<th>NNG alone</th>
<th>NSG + NNG</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = \eta = 1$</td>
<td>$\lambda = 1.2, \eta = 1$</td>
<td>$\lambda = 1, \eta = 1.5$</td>
<td>$\lambda = 1.2, \eta = 1.5$</td>
</tr>
<tr>
<td>Excluded</td>
<td>3.0</td>
<td>3.9</td>
<td>5.0</td>
<td>5.8</td>
</tr>
<tr>
<td>Rentiers</td>
<td>0.9</td>
<td>1.0</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>Classical</td>
<td>64.1</td>
<td>64.0</td>
<td>57.8</td>
<td>57.7</td>
</tr>
<tr>
<td>Non-classical</td>
<td>32.0</td>
<td>31.1</td>
<td>36.0</td>
<td>35.3</td>
</tr>
</tbody>
</table>

Four main outcomes can be highlighted:
1) Both North-South and North-North globalization increase the number of excluded.
2) Both North-South and North-North globalization increase the number of rentiers.
3) Both NSG and NNG lessen the number of classical households.
4) The number of non-classical increases because of the decrease in redistribution (NNG).

6.3.2. Working time

The theoretical model does not permit to conclude on the impact of globalization on the total working time because of several opposite effects linked to the changes in factor payments. In contrast, it is possible to calculate the changes in the total working time from our simulations. The precise results are available from the authors upon request. They show that:
1) Globalization lessens the working time along its extensive margin, i.e., the number of working households diminishes. This directly stems from the increase in the numbers of excluded and rentiers.
2) Globalization increases the working time along its intensive margins: working households work longer. This essentially comes from the non-classical households who work longer because of the decrease in $w_L$ and in redistribution.
3) All in all, the total working time slightly decreases.

7. Discussion and conclusion

From a model in which households differ in their skill and capital endowments, we have shown that labour supply behaviours generate four social groups, i.e., the excluded, the rentiers, the classical and the non-classical.

We have subsequently introduced globalization by making a distinction between North-North and North-South globalization. NNG creates corporate tax competition whereas NSG increases the return to capital and skill at the expense of the payment for simple labour. The combination of both types of globalization modifies social segmentation. Both the space of
excluded and the space of rentiers increase. Consequently, globalization results in an enlargement of both extremities of the social space, namely, those who do not work because they are too poorly endowed with skill and capital to attain the minimal consumption, and those who do not work because their capital endowment is sufficiently high to discourage them working for the wage corresponding to their skill.

The increase in the space of exclusion can be illustrated by the increase in the poverty rate experienced by a number of advanced countries in the twenty last years. Note that the positive impact of the decline in the corporate tax rate upon exclusion due to lower redistribution can be counteracted (i) by higher levies on consumption or on labour incomes, and (ii) by an increase in public debt. This last possibility is however not sustainable in the long term.

One of the most notable predictions is the enlargement of the space of rentiers, thus the rise in their weight in the population. As rentiers do typically not belong to the lower class or the lower middle class, this prediction essentially concerns the upper class and upper middle class. In the XXth century, one of the prominent social changes in advanced economies was the vanishing of the rentiers (Piketty, 2003; Piketty & Saez, 2003). In addition, certain studies suggest that, despite the huge increase in the income share of the top of the income distribution in most advanced countries (Atkinson & Piketty, 2007), the class of rentiers is not yet reconstituted (Kopczuk & Saez, 2004). One can thereby ask the questions: What forms can take this recovery of the rentiers and is this prediction realistic? Firstly, the new rentiers can come from households whose return to capital has become high enough to incite them to retire earlier than expected. This behaviour results from the increase in their rents and the decrease in their skill (obsolescence, age-related decrease in dynamism etc.). Then, both the increase in the return to capital (rise in \( r \) and reduction of \( \tau \) ) and the decrease in the real unit wage \( w_i \) incite older workers to retire earlier if they possess a sufficient amount of capital.

Secondly, the new rentiers can be children from rich families (who have inherited or received bequests) whose efficiency level is not high enough to allow them having a high position in the professional hierarchy. They thus prefer to live of their rents rather than having a job they consider unattractive.

Thirdly, they can also be individuals who have accumulated a huge amount of capital because of very high pay at the beginning of their professional carrier due to both very high efficiency and very high working time. When their efficiency begins to decrease, they can choose to become rentiers because they possess a substantial amount of capital. This is the case of the so-called ‘golden boys’ of the nineties who became rentiers when their dynamism
and efficiency decreased because of age. Piketty (2014, p. 440) also highlights the case of successful entrepreneurs who decided to become rentiers during their lifetime.

Note that, as the group of rentiers comprises workers who retire early and children from enriched families, its increase is typically not immediate; it needs a certain time to occur.

Finally note that we do not analyse the intergenerational dynamics of social segmentation. Our model focuses on the direct impact of globalization upon the respective returns to capital and labour, and thereby on working time and the incentive to work of heterogeneous households who differ in their skill and capital endowments. In the longer term, this approach should be combined with a precise analysis of the impacts upon the formation and accumulation of skill (particularly education) and capital.

References


Antuofermo, M., Di Meglio E., 2012. 23 % of EU citizens were at risk of poverty or social exclusion in 2010. Eurostat, Statistics in focus — Population and social conditions, no 9/2012.


APPENDIX 1. The optimal working time

\[
\max_{c_i,t_i} u_i = b\left(c_i - \bar{c}\right)_+^{\alpha-1} + (1-t_i)_+^{\alpha-1} \quad \text{s.t.: } w_i t_i + (1-\tau)r_i + r_G \geq c \quad t_i \geq 0
\]

\[
u_i = b\left(w_i t_i + (1-\tau)r_i + r_G - \bar{c}\right)^{(\alpha-1)\alpha} + (1-\tau)^{(\alpha-1)\alpha}
\]

\[
\frac{\partial u_i}{\partial t_i} = \frac{\sigma-1}{\sigma} b\left(w_i t_i + (1-\tau)r_i + r_G - \bar{c}\right)^{-\alpha\sigma} w_i - \frac{\sigma-1}{\sigma} (1-t_i)^{-\sigma\alpha} = 0
\]

Hence:
\[
t_i = \max \left\{ \frac{\left(bw_i\right)^\sigma - (1-\tau)r_i - r_G + \bar{c}}{w_i + bw_i} \frac{\left(bw_i\right)^\sigma - r_i + \tau(r_i - \bar{r}) + \bar{c}}{w_i + bw_i} \right\} 0
\]

APPENDIX 2. Analysis of the working time function

Proof of Lemma 2.

\[
t_i = t_i (w_i) = \frac{\left(bw_i\right)^\sigma - (1-\tau)r_i - r_G + \bar{c}}{w_i + (bw_i)^\sigma}, \quad \left(bw_i\right)^\sigma - r_i + \tau(r_i - \bar{r}) \geq \bar{c}
\]

\[
t_i \underset{w_i \rightarrow \infty}{\longrightarrow} 1; \quad w_i + (1-\tau)r_i + r_G = \bar{c} \Rightarrow t_i = 1
\]

\[
\frac{\partial t_i}{\partial w_i} = \frac{\sigma b^\sigma w_i^{\sigma-1} \left(w_i + (bw_i)^\sigma\right) - (1 + \sigma b^\sigma w_i^{\sigma-1}) \left((bw_i)^\sigma - (1-\tau)r_i - r_G + \bar{c}\right)}{\left(w_i + (bw_i)^\sigma\right)^2}
\]

\[
\frac{\partial t_i}{\partial w_i} = \frac{\left(\sigma - 1\right) \left(bw_i\right)^\sigma + \left(1 + \sigma b^\sigma w_i^{\sigma-1}\right) \left((1-\tau)r_i + r_G - \bar{c}\right)}{\left(w_i + (bw_i)^\sigma\right)^2}
\]

1) \(1-\tau)r_i + r_G - \bar{c} > 0 \Rightarrow \partial t_i / \partial w_i > 0

2) \(1-\tau)r_i + r_G - \bar{c} < 0 \Rightarrow \partial t_i / \partial w_i < 0 \iff (\sigma - 1)b^\sigma w_i^{\sigma-1} \geq \sigma b^\sigma w_i^{\sigma-1} + 1) \left(\bar{c} - (1-\tau)r_i - r_G\right).

Hence:
\[
\frac{\partial t_i}{\partial w_i} \geq 0 \iff \frac{(\sigma - 1)b^\sigma w_i^{\sigma-1}}{b^\sigma w_i^{\sigma-1} + 1} \geq \bar{c} - (1-\tau)r_i - r_G.
\]

We denote:
\[
z(w_i) = \frac{(\sigma - 1)b^\sigma w_i^{\sigma-1}}{b^\sigma w_i^{\sigma-1} + 1}.
\]

\[\begin{align*}
\text{Figure A1. Function } z(w_i) & = \frac{b^\sigma (\sigma - 1)}{b^\sigma w_i^{\sigma-1} + w_i^{-\sigma}} \\
& \downarrow \\
& \begin{array}{c}
r_i \\
\bar{c} - r_i + \tau(r_i - \bar{r})
\end{array}
\end{align*}\]
Figure A1 depicts the position of function $z(w_i)$ in relation to $\bar{c} - (1-\tau)r_i - r_G$. 
$\Rightarrow \exists$ unique $\hat{w}(r_i)$ such that $w_i < \hat{w}(r_i, \bar{r}, \tau) \Rightarrow \partial t_i / \partial w_i < 0$ and $w_i > \hat{w}(r_i, \bar{r}, \tau) \Rightarrow \partial t_i / \partial w_i > 0$

Figure A2 depicts the working time $t_i$ depending on the wage $w_i = w_L + w_H t_i$ in the cases $(1-\tau)r_i + r_G < \bar{c}$ and $(1-\tau)r_i + r_G \geq \bar{c}$. In the first case (Figure A2a) the household works if $w_i + (1-\tau)r_i + r_G \geq \bar{c}$ and s/he is excluded if $w_i + (1-\tau)r_i + r_G < \bar{c}$ (see the analysis of function $t_i = t_i(w_i)$). When s/he works with a wage $w_i = w_L + w_H t_i$ lower than $\hat{w}_i$, the household is non-classical whereas s/he is classical in the case $w_i > \hat{w}_i$. In the second case (Figure A2.b) the household decides to live from its sole rents when the wage $w_i$ is smaller than $(1-\tau)r_i + r_G - \bar{c})^{1/\sigma}$, this value being the reservation wage of the household. If $w_i > ((1-\tau)r_i + r_G - \bar{c})^{1/\sigma}$, then the household works and is classical.

**APPENDIX 3. Proposition 1: The four social spaces**

The distribution of households between the classical and the non-classical depends on the sign of the derivatives $\partial t_i / \partial w_i$, $i=1,...,M$. In this respect, a first distinction can be made between
two cases, i.e., \((1-\tau)r_i + r_G - \bar{c} > 0\) and \((1-\tau)r_i + r_G - \bar{c} < 0\). In the first case, the household’s rents \((1-\tau)r_i + r_G\) are sufficient to cover the minimum consumption \(\bar{c}\). In the second case, the household must work to attain the minimum consumption \(\bar{c}\).

**Lemma A1:** Consider household \(i\) such that \((1-\tau)r_i + r_G - \bar{c} > 0\). Household \(i\) has a reservation wage \(w_i = b^{-1}\left((1-\tau)r_i + r_G - \bar{c}\right)^{\sigma}\) and it is classical if \(w_i > w_j\) and rentiers if \(w_i \leq w_j\).

**Proof.** Suppose that \((1-\tau)r_i + r_G - \bar{c} > 0\). Since \(t_i = \frac{(bw_i)^{\sigma} - \left((1-\tau)r_i + r_G - \bar{c}\right)}{(bw_i)^{\sigma} + \left((1-\tau)r_i + r_G - \bar{c}\right)}\), then \(t_i > 0 \iff \left((1-\tau)r_i - r_G + \bar{c}\right)\), and thus \(t_i > 0 \iff w_i > b^{-1}\left((1-\tau)r_i - r_G + \bar{c}\right)^{\sigma}\). Hence, \(w_i = b^{-1}\left((1-\tau)r_i - r_G + \bar{c}\right)^{\sigma}\) is household \(i\)’s reservation wage. If \(w_i > w_j\), then \(t_i > 0\) and \(\frac{\partial t_i}{\partial w_i} > 0\), i.e., household \(i\) is classical.

From inequalities \(w_i \leq w_j\) and \(w_i > w_j\), we can state the following

**Corollary.** Consider household \(i\) such that \((1-\tau)r_i + r_G > \bar{c}\). This household is rentier if \(k_i \geq \frac{\bar{c} - r_G + (bw_i)^{\sigma}}{(1-\tau)r}\) and classical if \(k_i < \frac{\bar{c} - r_G + (bw_i)^{\sigma}}{(1-\tau)r}\).

**Lemma A2:** Consider household \(i\) who is neither excluded nor a rentier. Then, this household is classical (non-classical) if \(k_i > (\leq) \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^{\sigma}w_i^{\sigma}}{(1-\tau)r\left(\sigma b^{\sigma}w_i^{\sigma-1} + 1\right)}\).

**Proof.** Household \(i\) is non-classical if: \(\frac{\partial t_i}{\partial w_i} < 0 \iff k_i < \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^{\sigma}w_i^{\sigma}}{(1-\tau)r\left(\sigma b^{\sigma}w_i^{\sigma-1} + 1\right)}\) and classical if: \(\frac{\partial t_i}{\partial w_i} > 0 \iff k_i > \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^{\sigma}w_i^{\sigma}}{(1-\tau)r\left(\sigma b^{\sigma}w_i^{\sigma-1} + 1\right)}\).

**Proof of Proposition 1.** Feature 1) derives from Lemma 1. Features 2) and 3) from Lemma A2, and feature 4 from Lemma A1 (corollary).

**APPENDIX 4. Limits and dimension of each social space**

1) **Space of exclusion**

The space of exclusion is below the line \(k_i = \frac{\bar{c} - (w_L + w_Hh_i)}{(1-\tau)r} - r_G\). In Figure 2, this line cuts the \(y\)-axis \((h_i = 0)\) at \(k_E = \frac{-r_G - w_L}{(1-\tau)r}\) and the \(x\)-axis \((k_i = 0)\) at \(h_E = \frac{\bar{c} - w_L - r_G}{w_H}\).
Hence, the space of exclusion dimension is \( S_E = \frac{(\bar{c} - w_L - \tau r \bar{K})^2}{2(1-\tau)rw_H} \).

2) Space of rentiers

The rentiers are such that \( k_i \geq \frac{b^\sigma (w_L + w_H h_i)^\sigma + \bar{c} - r_G}{(1-\tau)r} \).

In Figure 1, the curve \( k_i = \frac{\bar{c} - r_G + b^\sigma w_i^\sigma}{(1-\tau)r} \) cuts the y-axis \( (h_i = 0) \) at \( k_R = \frac{\bar{c} - r_G + b^\sigma w_L^\sigma}{(1-\tau)r} \)
and attains the value \( k_i = k_{max} \) for \( h_R = \frac{b^{-1}((1-\tau)k_{max} + r_G - \bar{c})^\sigma + \bar{c}}{(1-\tau)r} \).

The dimension of the space of the rentiers is \( S_R = k_{max}h_R - \int_0^{\bar{h}_R} \frac{b^\sigma (w_L + w_H h)^\sigma}{(1-\tau)r} dh = \frac{\bar{c} - r_G + b^\sigma w_L^\sigma}{(1-\tau)r} \).

3) Space of non-classical households

The non-classical households are such that: \( \frac{\bar{c} - r_G - w_i}{(1-\tau)r} \leq k_i < \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^\sigma w_i^\sigma}{\left(\sigma b^\sigma w_i^\sigma - 1\right)(1-\tau)r} \).

The curve \( k_i = \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^\sigma (w_L + w_H h_i)^\sigma}{\left(\sigma b^\sigma (w_L + w_H h_i)^\sigma - 1\right)(1-\tau)r} \) cuts the y-axis \( (h_i = 0) \) at \( k_C = k_E + \frac{w_L \left(\sigma b^\sigma w_L^\sigma - 1\right)}{(1-\tau)r \left(\sigma b^\sigma w_L^\sigma - 1\right)} \) and the x-axis \( (k_i = 0) \) at the value \( h_C \) which is the root of the equation in \( h: \left(\bar{c} - r_G\right)\sigma b^\sigma (w_L + w_H h)^\sigma - (\sigma - 1)b^\sigma (w_L + w_H h)^\sigma + (\bar{c} - r_G) = 0 \).

The dimension of the space of non-classical households \( S_{NC} \) is:
\[
S_{NC} = \int_0^{k_C} \frac{\bar{c} - r_G}{(1-\tau)r} - \frac{(\sigma - 1)b^\sigma (w_L + w_H h)^\sigma}{\left(\sigma b^\sigma (w_L + w_H h)^\sigma - 1\right)(1-\tau)r} dh
\]

4) Space of classical households

The dimension of the space of classical households is thus: \( S_C = k_{max}h_{max} - (S_R + S_{NC} + S_E) \).

Table A1 provides the limit values of each space (except \( h_C \)).
Table A1. The Spaces limit values

<table>
<thead>
<tr>
<th></th>
<th>$h_E$</th>
<th>$k_E$</th>
<th>$h_R$</th>
<th>$k_R$</th>
<th>$k_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\bar{c} - w_L - r_G$</td>
<td>$\bar{c} - r_G - w_L$</td>
<td>$(1 - \tau)r_{\text{max}} + r_G - \bar{c}$</td>
<td>$-bw_i$</td>
<td>$b\sigma w_i^\sigma + w_i$</td>
</tr>
</tbody>
</table>

APPENDIX 5. Impacts of $r$ and $\tau$ on $S_E$ and $S_R$ (proofs of Lemmas 3 and 4)

1) Space of exclusion: $S_E = \frac{(\bar{c} - \tau \bar{k} - w_L)^2}{2(1 - \tau)rw_H}$

$$\frac{\partial S_E}{\partial r} = -\frac{2(1 - \tau)w_H(\bar{c} - \tau \bar{k} - w_L)(\bar{c} - w_L)}{(2(1 - \tau)rw_H)^2} < 0$$ since $\bar{c} > \tau \bar{k} + w_L$ (the space of exclusion exists).

$$\frac{\partial S_E}{\partial \tau} = 2nw_H(\bar{c} - \tau \bar{k} - w_L)\frac{\bar{c} - w_L - (2 - \tau)\bar{k}}{(2(1 - \tau)rw_H)^2} < 0$$ as $\bar{c} < w_L + (2 - \tau)\bar{k}$ by assumption (A4 section 4.3).

Condition $\bar{c} < w_L + (2 - \tau)\bar{k}$ is realistic. Actually, $\bar{k}$ is the average capital income which accounts for about 30% of the average income in advanced countries. Provided that the highest possible level of $\tau$ is 50% and that the lowest possible share of simple labour in total income is 20%, then $w_L + (2 - \tau)\bar{k}$ is higher than 65% of the average income, which is typically above the poverty line and thus above $\bar{c}$.

2) The rentiers are such that $k_i \geq \frac{b\sigma (w_i + w_H h_i)^\sigma + \bar{c} - r_G}{(1 - \tau)r} - \frac{\bar{k}}{1 - \tau} = z(r)$.

As $\frac{\partial z}{\partial r} = -\frac{b\sigma (w_i + w_H h_i)^\sigma + \bar{c}}{(1 - \tau)} r^{-2} < 0$, an increase in $r$ augments the number of households that verify $k_i \geq z(r)$ and enlarges thereby the space of rentiers ($\partial S_R / \partial r > 0$).

The rentiers are such that $(bw_i)^\sigma - rk_i + \tau (rk_i - r\bar{k}) + \bar{c} = 0$. In the plan $(h_i, k_i)$, the curve $(bw_i)^\sigma - rk_i + \tau (rk_i - r\bar{k}) + \bar{c} = 0$ separates the rentiers from the non-rentiers. By differentiating we find $\frac{dk_i}{d\tau} = \frac{(k_i - \bar{k})}{(1 - \tau)}$. Since $k_i > \bar{k}$ for all the rentiers, an increase (decrease) in $\tau$ moves the curve $(bw_i)^\sigma - rk_i + \tau (rk_i - r\bar{k}) + \bar{c} = 0$ upwards (downwards) in the plan $(h_i, k_i)$, i.e., a decrease (increase) in the space of rentiers ($\partial S_R / \partial \tau < 0$).

APPENDIX 6. Proof of Lemma 5: Impact of wage upon the social spaces

1) The rentiers are such that $(bw_i)^\sigma \leq r (1 - \tau)k_i + \tau \bar{k} - \bar{c}$. Thus, an increase in $w_i$ reduces the space of the rentiers.

2) The excluded are such that $w_i < \bar{c} - r (k_i + \tau (\bar{k} - k_i))$. Thus, an increase in $w_i$ reduces the space of exclusion.
APPENDIX 7. Impacts of North-South globalization

1) NSG and the space of exclusion: 

\[ S_E = \frac{(\bar{\alpha} - \lambda^{\alpha_L} \tau \bar{r} \bar{k} - \lambda^{-\alpha_L} \bar{w}_{L})^2}{2(1-\tau)\lambda^{2\alpha_L} \bar{r} \bar{w}_H} = \frac{(\lambda^{\alpha_L} \bar{c} - \tau \bar{k} - \lambda^{-1} \bar{w}_{L})^2}{2(1-\tau)\bar{r} \bar{w}_H} \]

\[ \frac{\partial}{\partial \lambda} \left[ \lambda^{\alpha_L} \bar{c} - \tau \bar{k} - \lambda^{-1} \bar{w}_{L} \right] = -\alpha_L \lambda^{\alpha_L - 1} \bar{c} + \lambda^{-2} \bar{w}_{L} \]

Hence: \( \frac{\partial S_E}{\partial \lambda} > 0 \Leftrightarrow \lambda < (\bar{w}_L / \alpha_L \bar{c})^{(1/(1-\alpha_L))} \). Coefficient \( \lambda \) increases from the initial value 1.

In addition, the average income is higher than \( \bar{c} \) (Assumption 4) and \( \bar{w}_L = \alpha_L \times \)average income (because of the Cobb-Douglas technology). Hence: \( \bar{w}_L / \alpha_L \bar{c} > 1 \).

Consequently, \( \frac{\partial S_E}{\partial \lambda} > 0 \) at the outset of the NSG process, and \( \frac{\partial S_E}{\partial \lambda} < 0 \) from the time when \( \lambda \) attains \( (\bar{w}_L / \alpha_L \bar{c})^{(1/(1-\alpha_L))} \), i.e., an inverted-U relationship between \( S_E \) and \( \lambda \).

2) Impact of NSG on the Space of rentiers

In the space of households, the rentiers gathers all the households situated above the curve

\[ k = R(h) = \frac{b^\sigma \left( \lambda^{\alpha_L} \bar{w}_{L} + \alpha_L \bar{w}_{H} h \right)^\sigma - \tau \lambda^{\alpha_L} \bar{k} + \bar{c}}{(1-\tau)\lambda^{\alpha_L} \bar{r}} \]

\( \Leftrightarrow (1-\tau)\bar{r} k = b^\sigma \left( \lambda^{\alpha_L} \bar{w}_{L} + \alpha_L \bar{w}_{H} h \right)^\sigma - \tau \lambda^{\alpha_L} \bar{k} + \bar{c}. \)

If \( \frac{\partial k}{\partial \lambda} < 0 \forall h \in [0, h_{\text{max}}] \), then an increase in \( \lambda \) moves the curve \( R(h) \) downwards and enlarges the space of rentiers for all the values of human capital.

Remark: the dimension of the space of rentier can expand whereas the number of rentiers decreases. However, when the space of rentiers expands at each level of human capital \( h \in [0, h_{\text{max}}] \), then the number of rentiers increases.

By derivation:

\[ (1-\tau)\bar{r} \frac{\partial k}{\partial \lambda} = b^\sigma \left( \lambda^{\alpha_L - 1} \bar{w}_{L} + \alpha_L \bar{w}_{H} h \right)^{\sigma - 1} \left( \alpha_L (\sigma - 1) \lambda^{\alpha_L - 1} \bar{w}_{L} + \alpha_L (\sigma - 1) \lambda^{\sigma - 1} \bar{w}_{H} h \right) - \alpha_L \lambda^{\alpha_L - 1} \]

Hence: \( \frac{\partial k}{\partial \lambda} > 0 \Leftrightarrow b^\sigma \left( \lambda^{-1} \bar{w}_{L} + \bar{w}_{H} h \right)^{\sigma - 1} \left( 1 - \frac{\sigma}{\alpha_L (\sigma - 1)} \right) \lambda^{\alpha_L - 1} \bar{w}_{L} + \bar{w}_{H} h \) \( (\sigma - 1) \lambda^{\alpha_L} \bar{c} \geq \bar{c} \)

For the increase in \( \lambda \) to enlarge the space of rentiers for any \( h \), we must have \( \frac{\partial k}{\partial \lambda} < 0, \forall h \).

Let us place ourselves at the outset of NSG:

\( \lambda = 1 \Rightarrow (1-\tau) \bar{r} \frac{\partial k}{\partial \lambda} = b^\sigma \alpha_L (\sigma - 1) \left( \bar{w}_L + \bar{w}_H h \right)^{\sigma - 1} - \alpha_L \bar{c} \)

\( \bar{M}_L = \bar{w}_L \sum t_i h_i, \bar{r} \) being the households’ average working time, and \( \bar{w}_L \bar{M} = \alpha_L I \Rightarrow I / M = \bar{w}_L / \alpha_L \)

because of the Cobb-Douglas technology. \( I / \bar{c} \Leftrightarrow \bar{w}_L \bar{r} > \alpha_L \bar{c}. \) Then \( \bar{M} < 1 \Rightarrow \bar{w}_L > \alpha_L \bar{c}. \)
\[
\lambda = 1: \frac{\partial k}{\partial \lambda} < 0, \ \forall h \iff \bar{w}_L + \bar{w}_H h < \frac{\sigma}{\alpha (\sigma - 1)} \bar{w}_L + \frac{\bar{c}}{b^\sigma (\sigma - 1) (\bar{w}_L + \bar{w}_H h)^{\sigma - 1}} \\
\lambda = 1, h = 0: \frac{\partial k}{\partial \lambda} < 0, \ \forall h \iff \left(1 - \frac{\sigma}{\alpha (\sigma - 1)}\right) \bar{w}_L < \frac{\bar{c}}{b^\sigma (\sigma - 1) \bar{w}_L^{\sigma - 1}}; \text{ always true because } \frac{\sigma}{\alpha (\sigma - 1)} > 0
\]

**Result:** At the outset of NSG (\( \lambda = 1 \)), the space of rentiers expands at each level of human capital and NSG creates more rentiers.

Subsequently, the progression of NSG has an ambiguous impact on the number of rentiers.

Denoting \( z(\lambda, h) = \left(\lambda^{-1} \bar{w}_L + \bar{w}_H h\right)^{\sigma - 1} \left(1 - \frac{\sigma}{\alpha (\sigma - 1)}\right) \lambda^{-1} \bar{w}_L + \bar{w}_H h\right)^{\lambda^{\alpha, \sigma}} \), we have:

\[
\frac{\partial k}{\partial \lambda} \leq 0 \iff z(\lambda, h) \leq \frac{\bar{c}}{b^\sigma (\sigma - 1)}. \quad \text{As} \quad \frac{\partial z}{\partial h} > 0, \text{ then that for any } \lambda \text{ the relation } z(\lambda, h) \leq \frac{\bar{c}}{b^\sigma (\sigma - 1)} \text{ can be analysed from Figure A3:}
\]

![Figure A3. The curve z(h)](image-url)

On Figure A3, (i) \( \frac{\partial k}{\partial \lambda} < 0 \) for the \( h \in \left[0, \hat{h}\right] \), and (ii) \( \frac{\partial k}{\partial \lambda} > 0 \) for the \( h \in \left[\hat{h}, h_{\text{max}}\right] \).

The frontier of the space of rentiers moves downwards and increases the number of rentiers and the dimension on the portion of the curve \( R(h) \) corresponding \( h \in \left[0, \hat{h}\right] \), and this frontier moves upwards and lessens the number of rentiers and the dimension on the portion of the curve \( R(h) \) with \( h \in \left[\hat{h}, h_{\text{max}}\right] \). Then, everything depends on the displacement of the curve \( z(h) \) when \( \lambda \) increases, i.e., of the sign of \( \frac{\partial z}{\partial \lambda} \). If \( \frac{\partial z}{\partial \lambda} > 0 \), the increase in \( \lambda \) reduces \( \hat{h} \) and the decrease in the dimension \( S_k \) due to the reduction of the space of rentiers above \( \hat{h} \) tends to prevail on its increase due to the expansion of the space beneath \( \hat{h} \). Unfortunately, the sign of \( \frac{\partial z}{\partial \lambda} \) is ambiguous, which does not permit to have a clear analysis of the changes in the number of rentiers throughout the NSG process.
APPENDIX 8. Inverted-U impact of NSG on the spaces of rentiers and excluded

Figure A4. Inverted-U relationship between $\lambda$ and $S_R$ and $S_E$